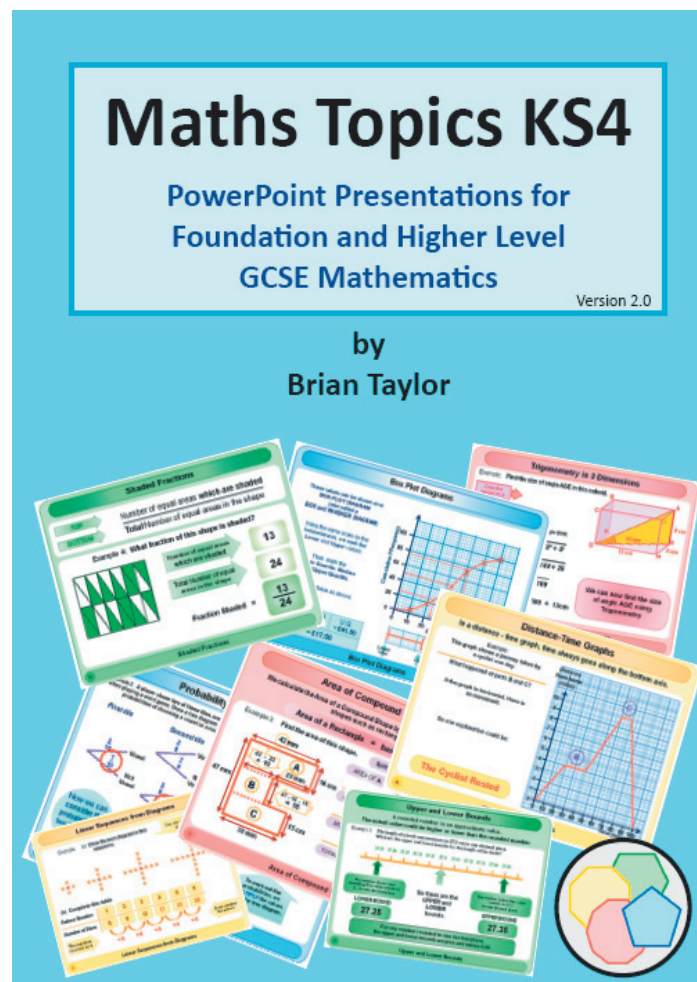


MATHS TOPICS

Powerpoint Presentations for
Foundation and Higher Level
GCSE Mathematics



A sample screen shot from each powerpoint
is given.

Subtracting Algebraic Fractions

To subtract algebraic fractions we use the same rules as for numerical fractions.

To subtract fractions we need to have the **BOTTOM** terms the **same**.

If the bottom terms of the fractions are not the same, then we have to make the fractions into equivalent fractions with the **same bottom term**.

Example 3: Subtract:

$$\frac{7g}{t} - \frac{3p}{y}$$

$$= \frac{7g \times y}{t \times y} - \frac{3p \times t}{y \times t}$$

$$= \frac{7gy}{ty} - \frac{3pt}{yt}$$

$$= \frac{7gy - 3pt}{ty}$$

- Step 1 → Multiply BOTH terms in FIRST fraction by BOTTOM TERM of SECOND fraction.
- Step 2 → Multiply BOTH terms in SECOND fraction by BOTTOM TERM of FIRST fraction.
- Step 3 → Subtract TOP TERMS. Bottom Term STAYS THE SAME.

A Adding Algebraic Fractions

Multiplying Terms

If we have different powers on the same letters, we add the powers.

Examples:

$$k^6 \times k^4 = k^{6+4} = k^{10}$$

$$n^3 \times n^5 \times n^{-2} = n^{3+5-2} = n^6$$

$$4 \times r^6 \times 7 \times r^8 \times r^{-5} = 4 \times 7 \times r^{6+8-5} = 28r^9$$

$$p^7 \times g^4 \times p^3 \times g^5 = p^{7+3} \times g^{4+5} = p^{10}g^9$$

$$f^{12} \times v^{-8} \times f^5 \times v^{-3} = f^{12+5} \times v^{-8-3} = f^{17}v^{-11}$$

A Collecting Terms and Simplifying

Algebra: Algebraic Fractions

Algebra: Collecting Terms and Simplifying

Completing the Square

This is a method which can be used to solve Quadratic Equations.

First the equation needs to be rearranged into the form:

$$ax^2 + bx + c = 0$$

If 'a' does not equal 1, then divide each term in the equation by 'a'.

Example 4:

By completing the square, Solve:

$$x^2 - 12x + 20 = 0$$

Step 1 → Halve the coefficient of x, and write it in a squared bracket like this:

$$(x - 6)^2$$

Step 2 → Expand the squared bracket:

$$(x - 6)^2 = x^2 - 12x + 36$$

Step 3 → Subtract the number terms:

$$20 - 36 = -16$$

Step 4 → Write your completed square:

$$x^2 - 12x + 20 = (x - 6)^2 - 16$$

Step 5 → Move the -16 to the other side (changing sign)

$$(x - 6)^2 = 16$$

Step 6 → Square root both sides

$$x - 6 = \pm 4 \quad \therefore x - 6 = 4 \quad \text{or} \quad x - 6 = -4$$

Step 7 → Add 6 to both sides

$$x = 4 + 6 \quad \text{or} \quad x = -4 + 6$$

$$x = 10 \quad \text{or} \quad x = 2$$

A Completing the Square

Conversion Graphs

These allow us to convert from one unit to another.

Example 1:

This graph converts between inches and centimetres.

Use the graph to convert 4 inches into cm.

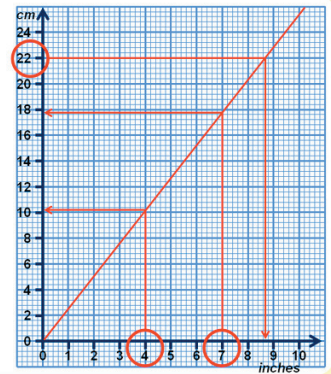
$$4 \text{ inches} = 10.2 \text{ cm}$$

Use the graph to convert 7 inches into cm.

$$4 \text{ inches} = 17.8 \text{ cm}$$

Use the graph to convert 22 cm into inches.

$$22 \text{ cm} = 8.7 \text{ inches}$$



A Conversion Graphs

Algebra: Completing the Square

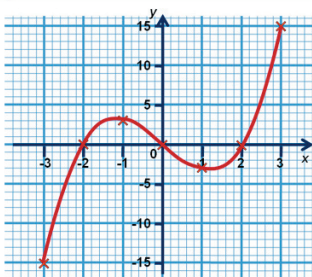
Algebra: Conversion Graphs

Drawing Cubic Graphs

Example 1: Draw the graph of $y = x^3 - 4x^2$ for x values between -3 and +3

x	-3	-2	-1	0	1	2	3
y	-15	0	3	0	-3	0	15

Plot the points on a graph.



Join points with a smooth curve.

A Cubic Graphs

Here is the rule for difference of squares written using letters:

$$a^2 - b^2 = (a + b)(a - b)$$

This is a difference of squares

Which can be written as a product

Algebraic Examples:

Factorise each expression

$$a^2 - 49 = a^2 - 7^2 = (a + 7)(a - 7)$$

$$p^2 - 144 = p^2 - 12^2 = (p + 12)(p - 12)$$

$$36n^2 - 16 = (6n)^2 - 4^2 = (6n + 4)(6n - 4)$$

$$25q^2 - 81r^2 = (5q)^2 - (9r)^2 = (5q + 9r)(5q - 9r)$$

A Difference of Squares

Algebra: Cubic and Reciprocal Graphs

Algebra: Difference of Squares

Distance-Time Graphs

In a distance - time graph, time always goes along the bottom axis.

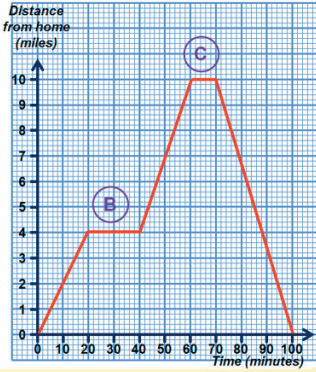
Example:
This graph shows a journey taken by a cyclist one day.

What happened at parts B and C?

If the graph is horizontal, there is no movement.

So one explanation could be:

The Cyclist Rested



Distance - Time Graphs

Equation and Graph of a Circle

A circle with radius r , and centre $(0, 0)$ has the equation:

$$x^2 + y^2 = r^2$$

Example 3:
Give the equation of this circle which has its centre at $(0, 0)$.

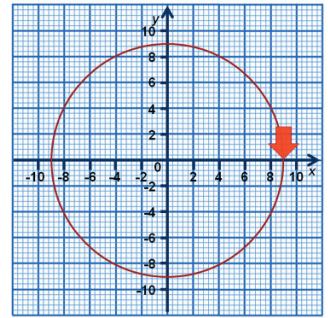
We can see that the circle crosses the x axis at $(9, 0)$

Therefore: $r = 9$

So: $r^2 = 81$

So the equation is:

$$x^2 + y^2 = 81$$



Equation and Graph of a Circle

Algebra: Distance - Time Graphs

Algebra: Equation and Graph of a Circle

Factorising is the opposite of Expanding Brackets.
We look for common factors and put brackets back into the expression.

Example 3: Factorise $56r^2 + 35rm$

First draw a pair of brackets.

Look to see if any of the numbers have a common factor.

$$7 (8r^2 + 5rm)$$

Look to see if any of the letters have a common factor.

7 is a factor of both 56 and 35, so write this outside the brackets.

$$7r (8r + 5m)$$

r is a common factor, so write this outside the brackets.

Divide each term by 7 and write these inside the brackets.

So the answer is:

$$7r(8r + 5m)$$

Divide each term inside the brackets by r .

Factorising Algebraic Expressions

If the sign in front of c is a - sign, then both signs in the factorised answer are different.

Factorising expressions of the form $x^2 + bx - c$

Example 3: Factorise $x^2 + 3x - 40$

The first terms have to multiply to give the x^2 term

First draw brackets

$$(x + 8)(x - 5)$$

So the first term in each bracket must be x

The last terms have to multiply to give the -40

Signs are different

So we list pairs of factors of 40

$$\begin{array}{l} 1 \times 40 \\ 2 \times 20 \\ 4 \times 10 \\ 5 \times 8 \end{array}$$

Then we choose the pair which subtract to give 3. As it's +3, the biggest number must go with the + sign

Factorising Quadratic Expressions

Algebra: Factorising Algebraic Expressions

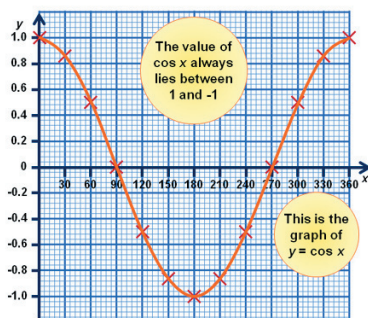
Algebra: Factorising Quadratic Expressions

Graph of $y = \cos x$

This table shows some values of $\cos x$ for angles between 0° and 360°

Angle (x°)	$\cos x$
0	1
30	0.866
60	0.5
90	0
120	-0.5
150	-0.866
180	-1
210	-0.866
240	-0.5
270	0
300	0.5
330	0.866
360	1

We can plot these values on a graph as shown below:



Graphs of Trigonometric Functions

Showing Inequalities on Graphs

We will show the method using the following examples:

Example 6:
Shade on this graph the region $y \leq 2x + 1$

First we draw the line:
 $y = 2x + 1$

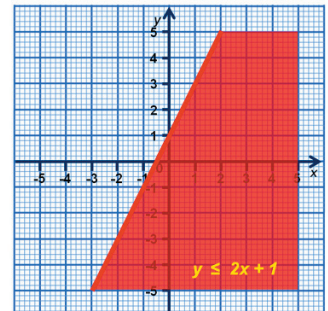
Now choose a point on one side of the line to see if it fits the inequality.

We will choose:
 $(1, 4)$

Here, $2x + 1$ gives us $2 + 1$, which is 3. This is less than the y number, but we wanted it to be more, so this point is on the wrong side of the line.

So, we shade BELOW the line.

SOLID LINE because the inequality is LESS THAN OR EQUAL to $2x + 1$



Showing Inequalities on Graphs

Algebra: Graphs of Trigonometric Functions

Algebra: Inequalities as Regions on Graphs

Inequalities on a Number Line

Example 6:
Show on a number line the values for x where $-2 < x \leq 7$

This means:
show all the values on the number line which are
MORE THAN -2 BUT LESS THAN OR EQUAL TO 7.

As the inequality does not include -2, we do not shade this circle.

As the inequality includes 7, we shade this circle.

A Inequalities on a Number Line

Solving two-step Inequalities

These require two steps to find the answer.

In these examples, the first step either is to **add** or **subtract**, and the second step is to either **divide** or **multiply**.

Example 5: Solve $\frac{h}{6} + 3 > 8$

Take 3 $\frac{h}{6} > 8 - 3$

Multiply by 6 $h > 5 \times 6$

$h > 30$

A Solving Inequalities

Algebra: Inequalities on a Number Line

Algebra: Inequalities: Solving

Two-step Linear Equations

These require two steps to find the answer.

In these examples, the first step either is to **add** or **subtract**, and the second step is to either **divide** or **multiply**.

Example 3: Solve $7k - 12 = 51$

Add 12 $7k = 51 + 12$

Divide by 7 $k = 63 \div 7$

$k = 9$

A Linear Equations

Linear Sequences from Diagrams

Example: (a) Draw the next diagram in this sequence. One star is added to each 'spoke', to give

(b) Complete this table

Pattern Number	1	2	3	4	5	6
Number of Stars	5	9	13	17	21	25

The numbers increase by 4.
 $+4$
 $+4$
 $+4$
 $+4$
 $+4$
So we continue the pattern.

A Linear Sequences from Diagrams

Algebra: Linear Equations

Algebra: Linear Sequences - Finding Terms and Rules

Finding the Midpoint of a Line Segment

If we know the coordinates of two points on a line, we can find the point which is half way between these points.

Example 3: Find the midpoint of the line joining $(-9, 7)$ and $(-1, 19)$.

x co-ordinate of mid point:
Add these together then divide by 2

$$\frac{(-9 + -1) \div 2}{= -10 \div 2}$$

$$= -5$$

y co-ordinate of mid point:
Add these together then divide by 2

$$\frac{(7 + 19) \div 2}{= 26 \div 2}$$

$$= 13$$

$(-9, 7)$ and $(-1, 19)$

So the midpoint of the line joining $(-9, 7)$ and $(-1, 19)$ is:

$(-5, 13)$

A Finding the Midpoint of a Line Segment

To multiply out a single pair of brackets, we **multiply each term inside the brackets by the term outside the brackets**.

Example 3: Multiply out $-2y(-6r + 8)$

First multiply this pair of terms

 $-2 \times -6 = 12$

$-2y(-6r + 8)$

Then multiply this pair of terms

 $-2 \times 8 = -16$

Then multiply the numbers

 $y \times r = yr$

Then include the letter

y

This gives us

$12yr$

So the answer is:

$12yr - 16y$

A Multiplying Out a Single Pair of Brackets

Algebra: Midpoint of a Line Segment

Algebra: Multiplying out a Single Pair of Brackets

There are various methods we can use to expand a pair of brackets.

(2) 'FOIL' Method

We can expand brackets by multiplying terms in this order:

Example 1: Multiply out $(x + 6)(x + 11)$

First
Outer
Inner
Last

$$(x + 6)(x + 11) = x^2 + 11x + 6x + 66$$

Combine the x terms: $11x + 6x$ gives $17x$
So the answer is:

$$x^2 + 17x + 66$$

A Multiplying Out a Double Pair of Brackets

Algebra: Multiplying out a Double Pair of Brackets

Plotting Co-ordinates in all Four Quadrants

Co-ordinates contain a pair of numbers in brackets which represent a point on a graph.

Examples:
Plot each of these co-ordinates on the grid.

$(-5, 7)$

$(-5, 7)$

First move 5 units LEFT
Then move 7 units UP

A Plotting Co-ordinates in all Four Quadrants

Algebra: Plotting Co-ordinates in all Four Quadrants

Drawing Quadratic Graphs

Example 2: Draw the graph of $y = 2x^2 - 4x + 3$ for x values between -2 and +4

x	-2	-1	0	1	2	3	4
y	19	9	3	1	3	9	19

Plot the points on a graph.

Join points with a smooth curve.

A Drawing Quadratic Graphs

Algebra: Quadratic Graphs

Multiple Step Rearrangements

Example 11: $P = h(4r - 3g)$

Make g the subject

$P = 4rh - 3gh$ → Expand the brackets

$P + 3gh = 4rh$ → Add 3gh to both sides

$3gh = 4rh - P$ → Subtract P from both sides

$g = \frac{4rh - P}{3h}$ → Divide both sides by 3h

A Rearranging Formulae

Algebra: Rearranging Formulae

Simplifying Algebraic Fractions containing Quadratics

Factorising the quadratic expressions in an algebraic fraction may help us to write the fraction more simply.

Example 4: Simplify: $\frac{6x^2 + x - 1}{2x^2 + 5x + 2}$

First factorise the quadratic expression on the top of the fraction

Then factorise the quadratic expression on the bottom of the fraction

Cancel terms which are the same

$$\frac{(3x - 1)(2x + 1)}{(2x + 1)(x + 2)} = \frac{3x - 1}{x + 2}$$

A Simplifying Algebraic Fractions containing Quadratics

Algebra: Simplifying Algebraic Fractions Containing Quadratics

Solving Equations by Trial and Improvement

Example 3: Show that the equation $x(x + 4) = 15$ has a solution between 2 and 3, and find the value of x to one decimal place.

We try different values of x:

Value for x	$x(x + 4)$	Working	Answer	High/Low Than 15
x = 2	$2 \times (2 + 4)$	2×6	12	LOW
x = 3	$3 \times (3 + 4)$	3×7	21	HIGH
So there must be a value of x BETWEEN 2 and 3				
We now try values with ONE decimal place				
x = 2.5	$2.5 \times (2.5 + 4)$	2.5×6.5	16.25	HIGH
x = 2.3	$2.3 \times (2.3 + 4)$	2.3×6.3	14.49	LOW
x = 2.4	$2.4 \times (2.4 + 4)$	2.4×6.4	15.36	HIGH
So there must be a value of x BETWEEN 2.3 and 2.4				
We now try HALF WAY BETWEEN 2.3 and 2.4 to see which value the solution is nearest to.				
x = 2.35	$2.35 \times (2.35 + 4)$	2.35×6.35	14.9225	LOW
2.35 is too LOW, so the solution must be ABOVE this:				
$x = 2.4$ (to 1 DP)				

NOTE: This step is important

A Solving Equations by Trial and Improvement

Algebra: Solving Equations by Trial and Improvement

Solving Quadratic Equations by Factorising

Some quadratic equations can be solved by factorising.

Example 3: Solve $x^2 - 11x - 12 = 0$

First factorise the quadratic expression

$$(x + 1)(x - 12) = 0$$

For the equation to equal 0, then either ...

$x + 1$ must equal 0 or $x - 12$ must equal 0

If $x + 1 = 0$ then $x = -1$ If $x - 12 = 0$ then $x = 12$

$x = -1$ or $x = 12$

A Solving Quadratic Equations by Factorising

Solving Quadratic Equations by using the Formula

Example 3: Solve $3x^2 + 5x - 8 = 0$

$a = 3$ $c = -8$

$b = 5$, so $-b = -5$, and $b^2 = 25$

Replace a, b and c with these values

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Calculate values in brackets and bottom of fraction.

$$x = \frac{-5 \pm \sqrt{25 - 4 \times 3 \times -8}}{2 \times 3}$$

Find the square root of 121

$$x = \frac{-5 \pm \sqrt{121}}{6}$$

So $x = \frac{-5 + 11}{6}$ or $x = \frac{-5 - 11}{6}$

$x = \frac{6}{6}$ or $x = \frac{-16}{6}$

$x = 1$ or $x = -2.67$

Note: the two minus signs in the brackets become a PLUS

A Solving Quadratic Equations by using the Formula

Algebra: Solving Quadratic Equations by Factorising

Algebra: Solving Quadratic Equations by using the Formula

Solving Simultaneous Equations by Elimination

We need to have the same number in front of one of the letters in both equations.

Example 6: Solve these simultaneous equations:

$$14x - 2y = 21 \quad \dots (1)$$

$$6x + 5y = 50 \quad \dots (2)$$

First we label the equations (1) and (2)

Now we will make the numbers in front of y the same in both equations.

$$70x - 10y = 105 \quad \dots 5 \times (1)$$

$$12x + 10y = 100 \quad \dots 2 \times (2)$$

To do this, first we multiply every term in equation (1) by 5, (because this is the number in front of the y in equation 2).

$$82x = 205$$

$$x = 205 \div 82$$

Next we multiply every term in equation (2) by 2, (because this is the number in front of the y in equation 1).

$$x = 2.5$$

Now we ADD corresponding terms to eliminate y.

$$14 \times 2.5 - 2y = 21 \quad \dots (1)$$

$$35 - 2y = 21$$

$$-2y = -14$$

$$y = -14 \div -2$$

$$y = 7$$

Then divide both sides by 2.

Now substitute for $x = 2.5$ into (1).

Take 35 from both sides

Divide both sides by -2 to give y.

A Solving Simultaneous Equations by Elimination

Solving Simultaneous Equations by Substitution

Example 5: Solve these simultaneous equations:

$$y = 2x - 9 \quad \dots (1)$$

$$5x - 2y = 27 \quad \dots (2)$$

First we label the equations (1) and (2).

Replace y in equation (2) with $2x - 9$ from equation (1).

$$5x - 2(2x - 9) = 27 \quad \dots (2)$$

Expand the brackets.

$$5x - 4x + 18 = 27$$

Combine the x terms.

$$x + 18 = 27$$

Take 18 from both sides.

$$x = 27 - 18$$

$$x = 9$$

Now substitute for $x = 9$ into (1).

$$y = 2 \times 9 - 9 \quad \dots (1)$$

Work out the multiplication.

$$y = 18 - 9$$

Subtract the value to give y.

$$y = 9$$

A Solving Simultaneous Equations by Substitution

Algebra: Solving Simultaneous Equations by Elimination

Algebra: Solving Simultaneous Equations by Substitution

Solving Simultaneous Equations Graphically

We draw the graph of each equation, and the co-ordinates of the crossing point tell us the x and y values which solve both equations.

Example 4: Find graphically the solution of these simultaneous equations:

$$y = 2x - 6$$

$$y = x - 2$$

Now we draw the graph of:

$$y = x - 2$$

Choose some x values take 2 This gives the y values

-8	→	-10
0	→	-2
10	→	8

Plot these pairs of values as points on the graph.

The crossing point of the lines give us the x and y values which are the solutions to the equations.

$x = 4$
 $y = 2$

A Solving Simultaneous Equations Graphically

Diagonal Graphs

Gradient of a Straight Line Graph (1)

If we stand on a point in the line and move **one** unit to the **right**, the **gradient** is the number of units we have to move **up, or down**, to get back onto the line.

We have to move **4 units DOWN** to get back onto the line

Gradient = -4

Lines which **rise** have **positive** gradients.

Lines which **fall** have **negative** gradients.

A Diagonal Graphs

Algebra: Solving Simultaneous Equations Graphically

Algebra: Straight Line Graphs

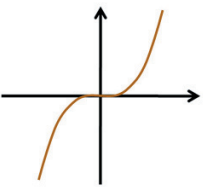
Stretch Parallel to the y axis

We stretch a graph parallel to the y axis by multiplying the function by a constant.

In general terms: If $y = f(x)$
Then $y = af(x)$ stretches the graph parallel to the y axis by scale factor "a".

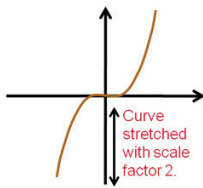
Example:

$y = x^3$



If we
**MULTIPLY THIS
FUNCTION
BY 2**
we get.

$y = 2x^3$



Curve stretched with scale factor 2.

A Transforming Graphs: Stretch Parallel to the y axis

Writing a Formula From Words

The key to doing this is to replace words and phrases with a single letter.

Example 3: The total cost, T , of hiring a power tool is a fixed price of £12, plus £3 per day.
Write a formula to find the total cost of hiring the tool for n days.

Write the connection in words

Total cost = £12 + £3 x Number of Days

Replace the words with letters

$T = 12 + 3 \times n$

Don't leave
units in your
formula

$T = 12 + 3n$

A Writing a Formula From Words

Algebra: Transforming Graphs

Algebra: Writing Formulae from Words

Estimate of Mean from Grouped Frequency Table

If we have grouped data, then we cannot work out an exact value for the mean. We use this method to calculate an ESTIMATE of the mean.

Example 2: This table gives the distances travelled to work by a number of people.

Distance in miles (m)	Number of people (Frequency)	Mid Value	Mid value x frequency
0 < m ≤ 2	7	1	7
2 < m ≤ 4	12	3	36
4 < m ≤ 6	6	5	30
6 < m ≤ 8	4	7	28
8 < m ≤ 10	3	9	27
Add the frequencies	32		128

Estimate of mean = $128 \div 32 = 4$ miles

Find the mid value for each range: (Do this by adding the pairs of numbers then dividing by 2).

Work out Mid Value x Frequency

Averages: Estimate of Mean from Grouped Frequency Table

An average is a value chosen to represent a set of data.

There are various ways of choosing an average.

MEAN

Example 3: Find the mean of:
105, 292, 345, 627, 319

Answer: Add the numbers:
 $105 + 292 + 345 + 627 + 319 = 1688$

There are **FIVE** numbers, so divide by 5:
 $1688 \div 5 = 337.6$

337.6 is the mean

To find the mean, add the numbers together and divide by how many numbers there are.

Averages: Mode, Median and Mean

Data: Averages - Finding an Estimate of the Mean

Data: Averages - Mode, Median, Mean and Range

Mean from a Frequency Table

If we have a frequency table, we can use this to work out the mean value of the data.

Example 1: This table gives the number of nails in some bags.

Number of Nails	Number of bags (Frequency)	Number of nails x Frequency
14	1	14
15	5	75
16	11	176
17	5	85
18	3	54
Add the frequencies	25	404

mean = $404 \div 25 = 16.16$ nails

This tells us the total number of bags

This tells us the total number of nails

Work out Number of nails x Frequency

Averages: Mean from a Frequency Table

Reading Bar Charts

In a bar chart, the length of each bar represents the frequency.

Example 2: This horizontal bar chart shows the favourite sports of a number of people.

Question b: How many people chose Tennis as their favourite sport?

This is found by reading the length of the bar.

2 People

Bar Charts

Data: Averages - Mean from a Frequency Table

Data: Bar Charts

Box Plot Diagrams

These values can be shown on a **BOX PLOT DIAGRAM** (also called a **BOX and WHISKER DIAGRAM**)

Using the same scale as the horizontal axis, we mark the Lower and Upper values.

Then mark the Lower Quartile, Median and Upper Quartile.

Finally join these as shown.

L.Q. = £24.00	MEDIAN = £33.50	U.Q. = £41.50
I.Q.R. = £17.50		

Box Plot Diagrams

Frequency Polygons

Frequency polygons allow us to display grouped data.

Example 2: The number of hours of overtime worked by the employees in a company were recorded one month. Draw a frequency polygon to illustrate this data.

Number of Hours (n)	Mid Value	Frequency
$0 \leq n < 2$	1	4
$2 \leq n < 4$	3	17
$4 \leq n < 6$	5	18
$6 \leq n < 8$	7	8
$8 \leq n < 10$	9	3

Draw the axes using suitable scales.

Plot each frequency against the mid-value of each range.

Join the points to produce a frequency polygon.

Frequency Polygons

Data: Cumulative Frequency and Box Plot Diagrams

Data: Frequency Polygons

Grouped Frequency Tables

If the data covers a wider range, it is easier to tally the data in groups.

Example: This list gives the height, in cm, for a number of people. Show the data in a Frequency Table.

150 169 142 164 159 174 162 152 149 156
 171 155 177 153 160 147 169 178 167 173
 151 162 150 157 181 164 159 174 163 152

We represent each number as a tally mark as shown.

We then add the tally marks and put the numbers in the Frequency column.

Height (h) cm	Tally	Frequency
$140 \leq h < 150$		3
$150 \leq h < 160$		11
$160 \leq h < 170$		9
$170 \leq h < 180$		6
$180 \leq h < 190$		1

Frequency Tables

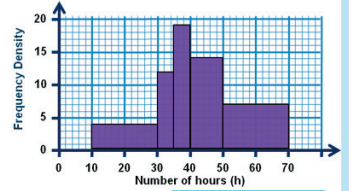
Histograms

In a BAR CHART, the BAR HEIGHT represents the frequency.

But, in a HISTOGRAM, the frequency is found from the AREA of the bar.

Example 2: This table shows number of hours worked one week by some employees. Show this information in a histogram.

Number of hours (h)	Frequency	Class Interval	Frequency Density
$10 \leq h < 30$	80	20	4
$30 \leq h < 35$	60	5	12
$35 \leq h < 40$	95	5	19
$40 \leq h < 50$	140	10	14
$50 \leq h < 70$	140	20	7



First we find the class interval: This is the gap between the hours.
 Next we do FREQUENCY ÷ CLASS INTERVAL To find the FREQUENCY DENSITY.
 Class Interval is the WIDTH OF THE BARS.
 Frequency Density is the HEIGHT OF THE BARS.
 So we draw the bars.

Histograms

Data: Frequency Tables and Grouped Frequency Tables

Data: Histograms

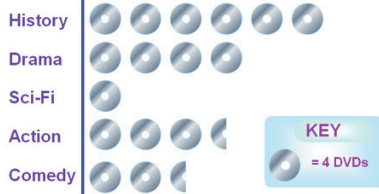
Pictograms

Pictograms use pictures to represent the frequency.

Example: This pictogram shows the number of DVDs in a collection.

Question b:
How many Action DVDs are there?

There are 3.5 pictures, and each one represents 4 DVDs. 3.5×4



14

Pictograms

Data: Pictograms

Drawing Pie Charts

To draw a pie chart, we need to work out the angles for each sector.

Example 2: This table shows the number of DVDs of different types on a shelf.

Type	Angle
Comedy	60°
Horror	80°
Sci Fi	35°
Cartoon	120°
Documentary	65°
	360°



First draw a circle, and draw a line from the centre to the outside

Then draw each angle using a protractor

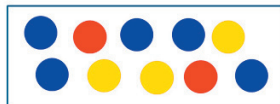
Pie Charts

Data: Pie Charts

Probabilities adding up to 1

For a given situation the probabilities of all the events which can happen will add up to 1.

Example: Give the probability of choosing each coloured disc from this box.



There are: 5 blue discs, 2 red discs, 3 yellow discs. There are 10 discs to choose from.

$P(\text{Blue}) = \frac{5}{10}$, $P(\text{Red}) = \frac{2}{10}$, $P(\text{Yellow}) = \frac{3}{10}$
 Adding these probabilities gives:

$$\frac{5}{10} + \frac{2}{10} + \frac{3}{10} = \frac{10}{10} = 1$$

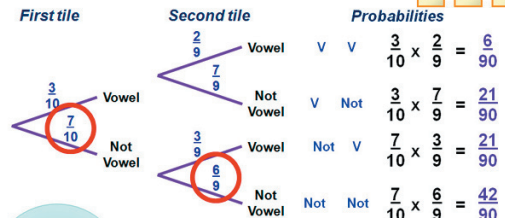
Probability: Probabilities adding up to 1

Data: Probability: Single Events

Probability: Tree Diagrams

Example 2: A player chose two of these tiles, one after the other when playing a word game. Draw a tree diagram to show all the probabilities of choosing a vowel or a non-vowel.

A H Y E G
L U T S N



Now we can consider the probabilities of two tiles being chosen.

The possible events which can happen.

To work out the probabilities, we MULTIPLY the values from the tree diagram.

Probability: Tree Diagrams

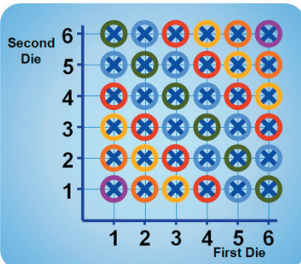
Data: Probability: Tree Diagrams

Probability: Listing Outcomes and Probability Space Diagrams

To find probabilities we need to know all the possible events which can happen.

Example 2: Two Dice are rolled. Give all the possible outcomes which can occur.

Another way to show these probabilities is to draw a probability space diagram:



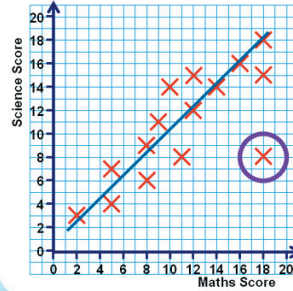
Score	Number of Ways	Probability
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

Probability: Probability Space Diagrams

Drawing a Scatter Diagram

Example 1: This table shows the scores, out of 20, obtained by a number of students in maths and science tests.

Pupil	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Maths Score	8	14	5	18	9	12	2	18	12	18	5	10	8	16	11
Science Score	6	14	7	18	11	15	3	8	12	15	4	14	9	16	8



We plot each pair of scores as points on a graph.

We can then draw a line of best fit.

This point is known as an outlier because it doesn't fit in with the pattern of the points.

This scatter diagram shows POSITIVE CORRELATION.

Scatter Diagrams: Types of Correlation

Data: Probability: Listing Outcomes and Probability Space Diagrams

Data: Scatter Diagrams

Stem and Leaf Diagrams

Example 3: This list gives the height in cm of 15 people:

178 175 192 184 180 173 177 191 181 183 193 192 181 175 182

Each number is split into two parts: A STEM and a LEAF.

Here, use the first two digits as the STEM...

And put each units digit as a LEAF.

Then put the LEAVES in order

Finally give a KEY which shows how the data is split.

17	8 5 3 7 5
18	4 0 1 3 1 2
19	2 1 3 2

STEM LEAF

17	3 5 5 7 8
18	0 1 1 2 3 4
19	1 2 2 3

17 | 3
means 173

Stem and Leaf Diagrams

Stratified Sample

Example 1:

The number of pupils in each year group of a school was as shown in the table.

100 pupils were to be chosen using a stratified sample. How many pupils from each year group should be chosen?

Year 7	Year 8	Year 9	Year 10	Year 11
100	60	90	110	140
100	60	90	110	140
500	500	500	500	500

The total number of pupils is 500, so we can write the fraction of pupils who are in each year group

So we need to find this fraction of 100 for each year group.

Year 7	$\frac{100}{500} \times 100 = 100 \div 500 \times 100$	20
Year 8	$\frac{60}{500} \times 100 = 60 \div 500 \times 100$	12
Year 9	$\frac{90}{500} \times 100 = 90 \div 500 \times 100$	18
Year 10	$\frac{110}{500} \times 100 = 110 \div 500 \times 100$	22
Year 11	$\frac{140}{500} \times 100 = 140 \div 500 \times 100$	28

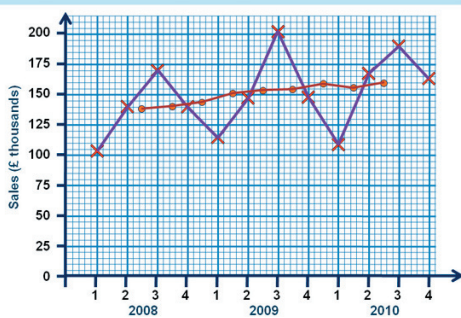
Stratified Sample

Data: Stem and Leaf Diagrams

Data: Stratified Sample

Time Series Graph

We can plot these Moving Averages on the Time Series Graph.



Moving Averages: 137 140 143 151 153 154 159 156 160

Time Series and Moving Averages

Two Way Tables

Two way tables present data which is split into different categories.

Example 2: This two way table shows the Lunch options for the pupils in a school. Complete the totals in the missing boxes.

	Year 10	Year 11	Totals
School Lunch	96	125	221
Packed Lunch	104	93	197
Totals	200	218	418

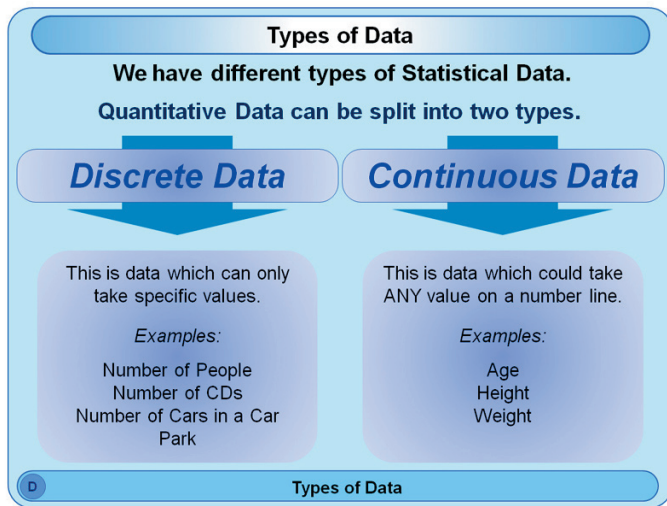
Year 10 / School $221 - 125 = 96$ Year 10 Total $96 + 104 = 200$

Year 11 / Packed $197 - 104 = 93$ Grand Total $221 + 197 = 418$

Two Way Tables

Data: Time Series and Moving Averages

Data: Two Way Tables



Data: Types of Data

Adding Decimals

To add decimals we need to keep the decimal points in line.

Example 5: Work out $298.7 + 6.876 + 59.48$

Write the numbers one on top of the other, with decimal points in line.

$$\begin{array}{r} \downarrow \\ 298.7 \\ + 6.876 \\ + 59.48 \\ \hline 365.056 \\ \hline \end{array}$$

1 2 2 1

Add the numbers in each column, starting from the right and working to the left.

- 6 = 6
So write this as shown.
- 7 + 8 = 15
So write this as shown.
- 7 + 8 + 4 + 1 = 20
So write this as shown.
- 8 + 6 + 9 + 2 = 25
So write this as shown.
- 9 + 5 + 2 = 16
So write this as shown.
- 2 + 1 = 3
So write this as shown.

Adding Decimals

Adding Whole Numbers

To add whole numbers we need to line up the numbers correctly.

Example 3: Work out $624872 + 51064 + 149757$

Write the numbers one on top of the other, with units digits in line.

$$\begin{array}{r} \downarrow \\ 624872 \\ + 51064 \\ + 149757 \\ \hline 825693 \\ \hline \end{array}$$

1 1 1 1 1

Add the numbers in each column, starting from the right and working to the left.

- 2 + 4 + 7 = 13
So write this as shown.
- 7 + 6 + 5 + 1 = 19
So write this as shown.
- 8 + 0 + 7 + 1 = 16
So write this as shown.
- 4 + 1 + 9 + 1 = 15
So write this as shown.
- 2 + 5 + 4 + 1 = 12
So write this as shown.
- 6 + 1 + 1 = 8
So write this as shown.

Adding Whole Numbers

Number: Adding Decimals

Number: Adding Whole Numbers

Best Value for Money

Method B

Example 4: Bubble bath is sold in two bottle sizes: SMALL offers 850 ml for 76p. LARGE offers 1.6 litres for £1.48. Which is best value for money? Show your working.

Quantity ÷ Price

Small		Large	
Quantity	Price	Quantity	Price
850 ml	76p	1.6 l	£1.48
Make Units the Same			
850 ml	76p	1600 ml	148p
850 ÷ 76		1600 ÷ 148	
11.18 ml per p		10.81 ml per p	

Small is Best Value

Best Value for Money

Changing a Recurring Decimal into a Fraction

If the recurring part *doesn't* start immediately after the decimal point, then we can express the decimal as a fraction by using methods as illustrated in the following examples:

Example 3: Convert $0.6\bar{5}8$ into a fraction

$$\begin{array}{r} \text{Let } y = 0.6\bar{5}8 \\ 10y = 6.\bar{5}8 \quad \leftarrow \text{Scale up the above (by multiplying both sides by 10) so that recurring part starts straight after decimal point} \\ 1000y = 658.\bar{5}8 \quad \leftarrow \text{Scale the above (by multiplying both sides by 100) to line up recurring parts} \\ 990y = 652 \quad \leftarrow \text{Subtract the two equations above} \\ y = \frac{652}{990} \quad \leftarrow \text{Divide both sides by 990} \\ = \frac{326}{495} \quad \leftarrow \text{Simplify} \end{array}$$

Check: do $326 \div 495$ on your calculator to see that it gives the correct decimal value.

Changing a Recurring Decimal into a Fraction

Number: Best Value for Money

Number: Changing a Recurring Decimal into a Fraction

Compound Percentage Reduction

METHOD 2: Using the Formula.

Again, the previous method is slow, particularly for a larger number of years, so we have a formula which allows us to find the total:

$$T = P \left(1 - \frac{R}{100}\right)^n$$

T = Total at end of time period

P = Original Amount

R = Percentage Rate of Interest

n = Number of Years

Example: Each year a car loses 21% of the value it was at the start of the year. If it was originally worth £9500, find its value after three years.

Put numbers into formula $T = 9500 \times \left(1 - \frac{21}{100}\right)^3$

Work out $21 \div 100$ $T = 9500 \times (1 - 0.21)^3$

Subtract $1 - 0.21$ $T = 9500 \times (0.79)^3$

Work out 0.79^3 $T = 9500 \times 0.493039$

Calculate the Answer

Value = £4683.87

This is the same as the previous example

Compound Percentage Reduction: Using the Formula

Converting a Percentage into a Fraction

Method:

write the percentage over 100, then simplify if possible

Examples: Convert each percentage into a fraction

$$\begin{array}{l} 41\% = \frac{41}{100} \\ 26\% = \frac{26}{100} = \frac{13}{50} \\ 5\% = \frac{5}{100} = \frac{1}{20} \end{array}$$

Converting between fractions, decimals and percentages

Number: Compound Interest and Compound Percentage Change

Number: Converting between Fractions, Decimals and Percentages

Estimating the Answer to a Calculation

This generally means that you should **FIRST** round the numbers to values which are easy to work out.
Rounding the numbers to one or two significant figures is often the best method.

Example 2: Estimate the answer to: $\frac{87.6 \times 7.9}{0.9^2 \times 30.8}$

Rounding each of these numbers to a sensible value gives: $\frac{90 \times 10}{1^2 \times 30}$

The top and bottom can now be easily worked out without a calculator. $\frac{900}{30}$

Now do the division to get an approximate answer of **30**

N Estimating the Answer to a Calculation

Lowest Common Multiple (LCM)

The Lowest Common Multiple of two numbers is the **LOWEST** number which is a **MULTIPLE** of **BOTH** NUMBERS.

Method 2: List the Multiples of each number

Example 2: Find the L.C.M. of 6 and 9.

List some Multiples of 6: 6, 12, 18, 24, 30, 36, 48

List some Multiples of 9: 9, 18, 27, 36, 45, 54, 63

Find the **SMALLEST** NUMBER in **BOTH** LISTS: **18**

HINTS: If you can't find a number in both lists, write more multiples. For the second list, you can stop when you've written a number which is already in the first list.

18 is the LCM of 6 and 9.

N Lowest Common Multiple (LCM)

Number: Estimating the Answer to a Calculation

Number: Factors, Multiples, HCF, LCM

Subtracting Fractions containing Mixed Numbers

If any fractions contains mixed numbers, we can first change any mixed numbers to top heavy fractions before subtracting the fractions.

Step 1: Change Mixed Numbers to Top Heavy Fractions.

Step 2: Multiply **BOTH** numbers in **FIRST** fraction by **BOTTOM NUMBER** of **SECOND** fraction.

Step 3: Multiply **BOTH** numbers in **SECOND** fraction by **BOTTOM NUMBER** of **FIRST** fraction.

Step 4: Subtract **TOP** NUMBERS. **Bottom Number STAYS THE SAME.**

Step 5: Change your answer to a mixed number if you are asked to.

Example 1: Subtract: $4\frac{2}{7} - 2\frac{5}{8}$

$$= \frac{30}{7} - \frac{21}{8}$$

$$= \frac{30 \times 8}{7 \times 8} - \frac{21 \times 7}{8 \times 7}$$

$$= \frac{240}{56} - \frac{147}{56}$$

$$= \frac{93}{56}$$

$$= 1\frac{37}{56}$$

N Fractions: Subtracting Fractions

Long Division

This method is to divide numbers **without** using a calculator.

Example 5: Divide 55718 by 26

Write the numbers like this:

Look at the first two digits to see how many times 26 goes into 55

It goes in 2 times: $26 \times 2 = 52$ So there's 3 left over

Now see how many times 26 goes into 37

It goes in 1 time: $26 \times 1 = 26$ So there's 11 left over

Now see how many times 26 goes into 111

It goes in 4 times: $26 \times 4 = 104$ So there's 7 left over

Now see how many times 26 goes into 78

It goes in 3 times: $26 \times 3 = 78$

26 $\overline{) 55718}$ $\begin{matrix} 2143 \\ \underline{52} \\ 37 \\ \underline{26} \\ 111 \\ \underline{104} \\ 718 \\ \underline{78} \\ 0 \end{matrix}$

N Long Division

Number: Fractions

Number: Long Division

Long Multiplication

This method is to multiply 2 or 3 digit numbers **without** using a calculator.

Example 5: Work out 689×43

Multiply the number on the top by the number on the side to get the number in each box

Then add the numbers in the boxes

	600	80	9	24000
40	24000	3200	360	3200
3	1800	240	27	360
				1800
				240
				27
				+
				29627

$43 = 40 + 3$ **Answer: 29627**

N Long Multiplication: Grid Method

Long Multiplication

This method multiplies two numbers **WITHOUT USING A CALCULATOR.**

Example 6: Work out 379×52

Finally we add the numbers to get the answer

$8 + 0 = 8$	379
$5 + 5 = 10$	$\times 52$
$7 + 9 + 1 = 17$	<hr/>
$8 + 1 = 9$	758
$1 = 1$	18950
	<hr/>
	19708
	11

$379 \times 52 = 19708$

N Long Multiplication: Traditional Method

Number: Long Multiplication - Grid Method

Number: Long Multiplication - Traditional Method

Money Questions

Questions involving money could need you to **add**, **subtract**, **multiply**, **divide**, or a **combination** of these operations.

Example 5:
A shopper had £38 in her pocket. She spent one quarter of her money. How much money did she have left?

Method

Divide by 4
to find a quarter of the amount.

Subtract £9.50
To find how much is left.

Amount Spent = $38 \div 4 = 9.50$

Amount Left = $38 - 9.50$

£28.50

N Money Questions

Number: Money Questions

Dividing by 10, 100, 1000 etc.

Whole Numbers

For whole numbers we just move the decimal point to the left.
In a whole number, the decimal point is to the right of the digits (It's usually not shown).

÷ by 10 → The decimal point moves **ONE** place to the left.

÷ by 100 → The decimal point moves **TWO** places to the left.

÷ by 1000 → The decimal point moves **THREE** places to the left.

Examples:

$7 \div 10 = 0.7$

$02 \div 100 = 0.02$

$006 \div 1000 = 0.006$

N Dividing by 10, 100, 1000 etc.

Number: Multiplying and Dividing by 10, 100, 1000 etc

Negative Numbers

We have some simple rules for calculating with Negative Numbers

Multiplying and Dividing

A **minus** number x (or ÷) a **minus** number gives a **PLUS** number.

Examples:

-5×-6

$= +30$

$= 30$

$-24 \div -6$

$= +4$

$= 4$

-17×-3

$= +51$

$= 51$

N Negative Numbers

Number: Negative Numbers

Triangular Numbers

Triangular Numbers form a triangular pattern of dots.

The first few are:

The numbers have a pattern as shown.
We can continue the pattern.
Another way of showing the pattern of dots is:

N Triangular Numbers

Number: Odd, Even, Prime and Triangular Numbers

Order of Operations

When we have calculations containing a number of operations, we have to do them in a certain order.

This list gives the order in which the operations must be carried out:

Brackets
Powers
Divide
Multiply
Add
Subtract

Example 7: Work out: $(42 - 4) \div (51 - 49)$

Work out each bracket first.

Keep the answers in the same position.

Now **DIVIDE** the numbers.

$38 \div 2$

19

N Order of Operations

Number: Order of Operations - BODMAS

Ordering Decimals

It's easiest to put decimals in order if we have the **same number of digits** after the decimal point.

Method

- Step 1** → Look at the decimals to find which one has the most number of digits after the decimal point.
- Step 2** → Add zeros to the other decimals so that they all have this number of digits after the decimal point.
- Step 3** → Put the numbers in order of size.

You can ignore the decimal points if this helps, but make sure you write them in your answer.

Example 3: Put these decimals in order of size from lowest to highest.

0.2 0.042 0.42 0.24 0.024

0.024 0.042 0.200 0.240 0.420

N Ordering Decimals

Number: Ordering Decimals

Finding an Original Amount

More difficult percentage questions involve finding an **original amount** after a percentage has either been added or taken from it.

The following examples will show how to find original amounts.

Example 1: A car was reduced in a sale by 10%.
If the sale price was £5850, what was the original price?

NOTE: 10% was calculated on the ORIGINAL PRICE and NOT on £5850, so the method is NOT to find 10% of 5850 and add this on.

Look at the percentages	$100\% - 10\% = 90\%$
This tells us that	5850 is 90% of the original price
Write this as an equation	90% of original price = 5850
Change 90% to a decimal	$0.9 \times \text{original price} = 5850$
Divide by 0.9	original price = $5850 \div 0.9$
Calculate the answer	original price = 6500

Original Price = £6500

N Percentages: Finding an Original Amount

Giving the Value of Digits in Numbers

Example 5: Give the value of the 6 in the number 61293.475

Put the number under the column headings:

Millions	Tens of Thousands	Hundreds	Units	Tenths	Thousandths
	Hundreds of Thousands	Thousands	Tens	Decimal Point	Hundredths
	↓	↓	↓	↓	↓
	6	1	2	9	3
				.	4
					7
					5

The 6 is in the **tens of thousands** column, so its value is:

60000

N Place Value

Number: Percentages

Number: Place Value

Powers: Fractional Powers

Rule: Fractional Powers refer to roots.

Number on the top of fraction is a normal power.

Example 2: Find $216^{\frac{4}{3}}$

$216^{\frac{4}{3}}$ means $(\sqrt[3]{216})^4 = 6^4 = 1296$

Do the CUBE ROOT first, then do the FOURTH POWER

Because $6 \times 6 \times 6$ equals 216

N Powers: Fractional Powers

The case where y is directly proportional to x²

Here, the method is similar, but the equation connecting x and y simply has x² instead of x.

Example: y is directly proportional to the square of x. When x = 5, y = 65.

(a) Find the equation connecting x and y.
(b) Find the value of y when x = 7.
(c) Find the value of x when y = 23.4.

Solution: (a) The equation connecting y and x has the type: $y = kx^2$

If x = 5, x² = 25

We find the value of k by doing $y \div x^2$

$k = \frac{65}{25} = 2.6$ so: $y = 2.6x^2$

(b) $y = 2.6x^2$
If x = 7, then x² = 49 so y = 2.6 x 49 $y = 127.4$

(c) $y = 2.6x^2$
so: $\frac{y}{2.6} = x^2$ so: $9 = x^2$
If y = 23.4, $\frac{23.4}{2.6} = x^2$ Square root $x = 3$

N Proportion: Direct Proportion

Number: Powers

Number: Proportion - Direct Proportion

The case where y is inversely proportional to x

Two sets of numbers are in **inverse proportion** if the pairs of numbers always give the same answer when they are multiplied.

Example 2: y is inversely proportional to x. When x = 3, y = 12.

(a) Find the equation connecting x and y.
(b) Find the value of y when x = 9.
(c) Find the value of x when y = 14.4.

Solution: (a) The equation connecting y and x has the type: $y = \frac{k}{x}$

We find the value of k by doing $y \times x$

$k = 12 \times 3 = 36$ so: $y = \frac{36}{x}$

(b) $y = \frac{36}{x}$
We are given x = 9, so $y = 36 \div 9$ $y = 4$

(c) $y = \frac{36}{x}$
so: $x = \frac{36}{y}$
We are given y = 14.4, so: $\frac{36}{14.4} = x$ $x = 2.5$

N Proportion: Inverse Proportion

Writing a Ratio in the form n : 1

This is where we make the second number in the ratio into the number 1.

This allows us to easily see how many times more or less the first value is compared to the second.

METHOD Divide BOTH numbers in the ratio by the second NUMBER.

Example: Write the ratio 45p : £3.60 in the form n : 1

Change both numbers into pence

Divide both numbers by 360

This is the ratio written in the form n : 1

N Ratio: Writing a Ratio in the form n:1

Number: Proportion - Inverse Proportion

Number: Ratio

Reciprocals

Reciprocals of Decimals

The reciprocal of a decimal can also be found.
Again, we just do "1 divided by the number".

Examples:

Number	Reciprocal
0.5	$1 \div 0.5 = 2$
0.125	$1 \div 0.125 = 8$
0.05	$1 \div 0.05 = 20$

N Reciprocals: Decimals

Number: Reciprocals

Rounding to a given number of Decimal Places

The number of decimal places is the number of digits after the decimal point.

Example 6: Round 3.42971 to three decimal places.

This means that there has to be THREE digits after the decimal point in the answer.

3.42971
 $\downarrow \downarrow \downarrow$
 3.430

As this is a 9, we round the last two digits, (29) to 30.

This is 5 or over.

If this digit is FIVE OR MORE we round up.

$3.42971 = 3.430$ (to 3 DP)

N Rounding Numbers: Rounding to a given number of Decimal Places

Number: Rounding Numbers

Shaded Fractions

TOP

→

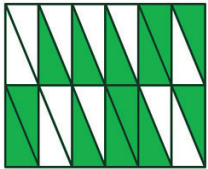
Number of equal areas which are shaded

BOTTOM

→

Total Number of equal areas in the shape

Example 4: What fraction of this shape is shaded?



Number of equal areas which are shaded → **13**

Total Number of equal areas in the shape → **24**

Fraction Shaded = $\frac{13}{24}$

N Shaded Fractions

Number: Shaded Fractions

Simple Interest

In SIMPLE INTEREST calculations, the interest is not added onto the original amount.

We have a formula to work out Simple Interest:

$$I = \frac{PRn}{100}$$

I = Amount of Interest

P = Original Amount

R = Percentage Rate of Interest

n = Number of Years

Example 2: Find the simple interest when £720 is invested for 8 years with interest rate 3.2%

Put the numbers into the formula

$I = 720 \times 3.2 \times 8 \div 100$

Work out the calculation

Interest = £184.32

N Simple Interest

Number: Simple Interest

Square Root

Finding the Square Root is the opposite of Squaring.

The easiest way to find the square root of a number is to use the SQUARE ROOT BUTTON on a calculator.

Calculators don't all work in the same way, so you need to find out how to operate the square root button on your own calculator.

Example 4: Evaluate $\sqrt{250}$

Use a calculator

$\sqrt{250}$

=

15.811388

So, the square root of 250 = **15.8** (to 3SF)

The symbol for square root is:

 $\sqrt{\quad}$

N Square Root

Number: Squares, Cubes and Roots

Converting a Standard Form Number to 'Ordinary' Form

This is the reverse of the previous method.

Example 1: Convert 3.7×10^6 to 'ordinary' form.

Step 1:
The POSITIVE power, 6, tells us move the decimal point 6 places to the RIGHT.

3.7×10^6

3700000

Step 2:
The 7 gives us ONE space.

Step 3:
So we need to Add FIVE zeros.

N Converting a Standard Form Number to 'Ordinary' Form

Number: Standard Form

Subtraction: Decimals

For decimals we simply keep the decimal points in line.

Example 1: Work out $98.7 - 5.63$

Write the numbers one on top of the other, with decimal points in line.

First subtract the HUNDRETHS DIGITS.

Because the top digit is smaller than the bottom one we have to borrow.

Reduce the 7 by 1, to make it into 6. Then put a 1 in front of the 0 to make it 10.

Then subtract the TENTHS DIGITS.

Then subtract the UNITS DIGITS.

Finally subtract the TENS DIGITS.

Add zeros so that you have a digit in each column

$$\begin{array}{r} 98.70 \\ -05.63 \\ \hline 93.07 \end{array}$$

$10 - 3 = 7$
 $6 - 6 = 0$
 $8 - 5 = 3$
 $9 - 0 = 9$

Subtraction

Surd Fractions: Rationalising the Denominator

We have a method for getting rid of surds which are on the bottom of a fraction.

Method: Multiply both the top and bottom of the fraction by the surd on the bottom of the fraction.

If we multiply the TOP and BOTTOM of a fraction by the same number, this does not alter the size of the fraction

Example 1: Simplify $\frac{8}{\sqrt{2}}$

Multiply top and bottom of fraction by $\sqrt{2}$

$\sqrt{2} \times \sqrt{2} = 2$
So replace this in the calculation.

$8 \div 2$ is 4, so this gives us:

$$= \frac{8 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$

$$= \frac{8 \times \sqrt{2}}{2}$$

$$= 4\sqrt{2}$$

Surd Fractions: Rationalising the denominator

Number: Subtraction

Number: Surds

Upper and Lower Bounds

A rounded number is an approximate value. The actual value could be higher or lower than the rounded number.

Example 3: The length of a book was measure as 27.3 cm to one decimal place. What are the upper and lower bounds for the length of the book?

27.25 27.26 27.27 27.28 27.29 27.3 27.31 27.32 27.33 27.34 27.35

Any number above (and including) this value rounds to 27.3 to one decimal place.

LOWER BOUND

27.25

So these are the **UPPER and LOWER** bounds.

Any number below this value rounds to 27.3 to one decimal place.

UPPER BOUND

27.35

For any number rounded to one decimal place, the upper and lower bounds are plus and minus 0.05

Upper and Lower Bounds

Writing a Number as a Product of Prime Factors

Example 3: Write 126 as a product of prime factors

126

Circle any factors which are prime numbers.

Hint: Try to find pairs of factors where one of them is a prime number

$126 = 2 \times 3 \times 3 \times 7$

This can also be written in terms of **POWERS** $2 \times 3^2 \times 7$

Writing a Number as a Product of Prime Factors

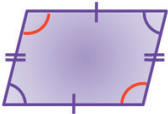
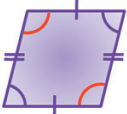


Number: Upper and Lower Bounds

Number: Writing a number as a product of Prime Factors

2D Shapes

Quadrilaterals

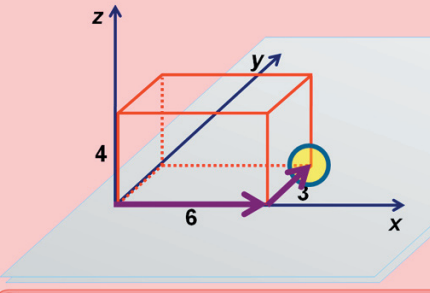
All quadrilaterals have 4 sides, and angles which add up to 360°. These are the different types:

Parallelogram	Rhombus	Trapezium
		
<p>Opposite angles are equal.</p> <p>Opposite sides are the same length and parallel.</p>	<p>Opposite angles are equal.</p> <p>All sides are the same length and there are two pairs of parallel sides.</p> <p>An isosceles trapezium has two equal angles and a pair of equal sides.</p>	<p>All angles could be different.</p> <p>One pair of parallel sides. All sides could be different.</p>
		<h4 style="margin: 0;">Isosceles Trapezium</h4> 

S 2D Shapes: Quadrilaterals

Three Dimensional Co-ordinates

(6 , 3 , 0)



This cuboid has base 6, height 4 and depth 3 units

The co-ordinates of any corner of the cuboid are written in the form (x, y, z)

Here's another corner

S Three Dimensional Co-ordinates


Shape: 2D Shapes

Shape: 3D Co-ordinates

Angles in Parallel Lines

INTERIOR ANGLES ADD UP TO 180°

Interior angles are on the same side of the transversal.




Angles A and D are INTERIOR

A + D = 180°

Angles B and C are INTERIOR

B + C = 180°

Example: Find the size of angle W, giving a reason for your answer.



$180 - 57 = 123$

W = 123°

Reason:
W and 57 ARE INTERIOR ANGLES, and Interior Angles add up to 180°.

S Angles: Angles in Parallel Lines

Angles: Measuring Angles

We measure angles using a protractor.

Method for using a protractor

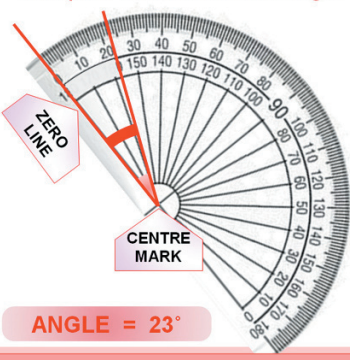
Position the protractor with the CENTRE MARK at the corner of the angle.

Now turn the protractor so that one of the lines of the angle is on a ZERO LINE.

Follow the scale from ZERO until you get to the other line of the angle.

0 is on the OUTSIDE SCALE, so use the numbers on that scale. IGNORE THE OTHER SCALE.

Example 4: Measure the shaded angle.



ANGLE = 23°

S Angles: Measuring Angles

Shape: Angles: Calculating Angles

Shape: Angles: Measuring Angles

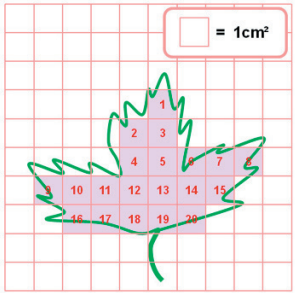
Estimating Area by Counting Squares

To do this, count all the squares in which more than half are in the shape.

Example:
Find an approximate area for this leaf.

Count the squares which have more than half the square in the leaf.

Area = 20 cm²



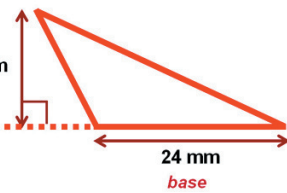
S Area by Counting Squares

Area of a Triangle

If we have two lengths at right angles, then we calculate the Area of a Triangle by using the formula:

Area = base x perpendicular height ÷ 2

Example 3:



Area = 24 x 9 ÷ 2

= 108 mm²

Don't forget ÷ 2 for a triangle

S Area of a Triangle: Two lengths at right angles

Shape: Area by Counting Squares

Shape: Area of a Triangle

Area of Compound Shapes

We calculate the Area of a Compound Shape by splitting this up into easier shapes such as rectangles:

Area of a Rectangle = base x height

Example 3: Find the area of this shape. Split the shape up into rectangles.

AREA OF A	We know the base and height, so: Area of A = 42×16 = 672
AREA OF B	We need to find the missing lengths: Area of B = 19×16 = 304
AREA OF C	We know the base and height, so: Area of C = 33×15 = 495
TOTAL AREA	= 672 + 304 + 495 = 1471 mm²

Area of Compound Shapes

Area of a Trapezium

We calculate the Area of a Trapezium by using the formula:

Area = (top + base) x perpendicular height ÷ 2

Example 2:

$$\text{Area} = (5.6 + 6.4) \times 2.4 \div 2$$

$$= 12 \times 2.4 \div 2$$

$$= 14.4 \text{ cm}^2$$

Area of Quadrilaterals: Trapezium

Shape: Area of Compound Shapes - Rectangles

Shape: Area of Quadrilaterals

Bearings

A bearing gives us the **direction** of one place from another.

To find the bearing of a place B FROM a place A, we do the following:

- Draw a line joining A and B.
- Draw a NORTH line at A.
- Find the **CLOCKWISE** angle between these two lines.
- Write this angle as a **3-digit** number.

Example 2: Find the bearing of G from F.

The bearing is the shaded angle

Angles in a full turn add up to 360°

Bearing = 360 - 246

360 - 246 = 114

Bearing of G from F = 114°

Bearings

Opposite angles of a cyclic quadrilateral add up to 180°

A cyclic quadrilateral is one where all four corners touch the circumference of the circle.

Opposite angles will always add up to 180°.

$$a + c = 180^\circ$$

$$b + d = 180^\circ$$

Circle Theorems

Shape: Bearings

Shape: Circle Theorems

Area and Perimeter of Compound Shapes: Circles

We calculate the Area and Perimeter of a Compound Shape by splitting this up into easier shapes such as squares, rectangles and circles.

Example 2: This running track has straight sides of length 24m, and ends which are semi circles of radius 8. Find the area and perimeter of the running track.

Use $\pi = 3.14$

Perimeter

Perimeter = 2 Lengths of Rectangle + Circumference of Circle

$$= 2 \times \text{base} + 2 \times \pi \times r$$

$$= 2 \times 24 + 2 \times 3.14 \times 8$$

$$= 48 + 50.24$$

$$= 98.24$$

Perimeter = 98.2 m (to 3sf)

Area and Perimeter of Compound Shapes: Circles

Finding the Area of a Sector

The sector area is a fraction of the area of the circle

We have a formula to work this out:

$$\text{Sector Area} = \frac{\text{Sector Angle}}{360} \times \pi r^2$$

Example 2: Find the area of sector b.

$$\text{Sector Area} = \frac{\text{Sector Angle}}{360} \times \pi r^2$$

$$\text{Sector Area} = \frac{100}{360} \times 3.14 \times 2.5^2$$

$$= 100 \div 360 = 0.278$$

$$\text{Sector Area} = 0.278 \times 3.14 \times 2.5^2$$

$$\text{Sector Area} = 5.45575$$

Sector Area = 5.46 m² (to 3sf)

Circles: Finding the Area of a Sector

Shape: Circles - Compound Areas and Perimeters

Shape: Circles - Finding Arc Length and Areas of Sectors

Area of a Circle

To find the area of a circle we need the **radius**.

We have a formula to work out the area (we can write this in two similar ways):

Area = pi times radius times radius or **Area = pi times radius squared**

$A = \pi \times r \times r$

eg: Find the area of a circle with radius of 2.8 m.

$A = \pi \times r \times r$

$A = 3.14 \times 2.8 \times 2.8$

$A = 24.6176$

$A = 24.6 \text{ m}^2$ (to 3sf)

$A = \pi \times r^2$

eg: Find the area of a circle with radius of 32 cm.

$A = \pi \times r^2$

$A = 3.14 \times 32^2$

$A = 3215.36$

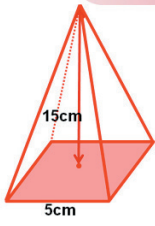
$A = 3220 \text{ cm}^2$ (to 3sf)

S Circles: Area of a Circle

Volume of a Pyramid

We have a formula to find the Volume of a Pyramid.

Volume = $\frac{1}{3}$ x Area of Base x height



Example 1:
Find the volume of a square based pyramid of height 15cm, and base sides of length 5cm.

Volume = $5 \times 5 \times 15 \div 3$

= $375 \div 3$

= 125 cm^3

Volume = 125 cm^3

S Pyramids: Volume

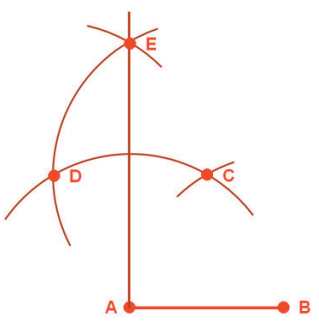
Shape: Circles - Parts, Lengths, Area

Shape: Cones and Pyramids - Surface Area and Volume

Constructing a 90° Angle at the End of a Line

Method

- Place compasses at **A** and draw an arc as shown.
- Place compasses at **B** and draw an arc as shown.
- Place compasses at **C** and draw an arc as shown.
- Place compasses at **D** and draw an arc as shown.
- Join points **A** and **E** to construct the 90° angle.
- Set your compasses to **THE LENGTH OF THE LINE** and keep them at this setting.



S Constructions: Constructing a 90° Angle at the End of a Line

Converting between units of Length, Area and Volume

LENGTH

1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m

AREA

1 cm² = 100 mm²
1 m² = 10000 cm²
1 km² = 1000000 m²

VOLUME

1 cm³ = 1000 mm³
1 m³ = 1000000 cm³
1 km³ = 1000000000 m³

Example 7:
A box has a volume of 290000mm³. Give this in cm³.

Divide by 1000

➔

Volume = $290000 \div 1000 \text{ cm}^3$

Volume = 290 cm³

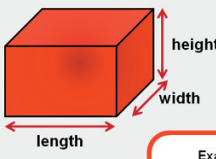
S Converting between units of Length, Area and Volume

Shape: Constructions

Shape: Converting between units of Length, Area and Volume

Volume of a Cuboid

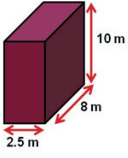
In a cuboid, all three sides could be different lengths.



We have a formula to work out the volume of a cuboid:

Volume = length x width x height

Example 2: Find the volume of a cuboid with length 2.5cm, width 8cm and height 10cm.



Volume = length x width x height

= $2.5 \times 8 \times 10$

Volume = 200 cm³

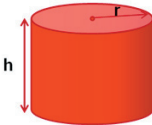
S Cuboids: Volume

Volume of a Cylinder

We have a formula to find the Volume of a Cylinder.

Volume = Area of base x height

Volume = $\pi \times r^2 \times h$



Example 2: Find the volume of a cylinder with radius of 8.2m and height 5.5m.

Volume = $3.14 \times 8.2^2 \times 5.5$

= $3.14 \times 67.24 \times 5.5$

= 1161.2348 m^3

Use $\pi = 3.14$

Volume = 1160 m³ (to 3sf)

S Cylinders: Volume

Shape: Cubes and Cuboids: Surface Area & Volume

Shape: Cylinders: Surface Area & Volume

Calculating the Mass

Example 1: A plastic shape of density 1.08g/cm^3 has a volume of 225cm^3 . Calculate the mass of the shape.

$$\text{Mass} = \text{Density} \times \text{Volume}$$

Put numbers into formula

$$\text{Mass} = 1.08 \times 225$$

Work out 1.08×225

$$\text{Mass} = 243\text{ g}$$

Density: Calculating the Mass

Dimensions

We can tell whether a formula represents a length, area, volume (or none of these) by looking at the **DIMENSIONS** of the formula.

We can use the method as shown in the following examples:

Example 3: Does this formula represent a **LENGTH**, **AREA** or **VOLUME**?

$$\pi r^2 h$$

Ignore any constants (these are numbers, including π), which are either multiplying, dividing, adding or subtracting the letters in the formula.

$$r^2 h$$

Each letter in the formula represents one dimension, so: Replace each letter with the letter **L** (to represent length).

$$L^2 \times L$$

$L^2 \times L$ (or L^3) 3 dimensions **VOLUME**

So this formula represents a **VOLUME**

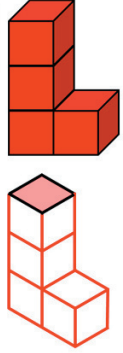
Dimensions

Shape: Density, Mass and Volume

Shape: Dimensions

Drawing 3D Shapes on Isometric Paper

Example: Draw this 3D solid on isometric paper.



Note: We have to draw the solid like this.

- Start by drawing the top.
- The solid is 3 units high, so we will draw the sides.
- Now draw these diagonal lines.
- Then draw these vertical lines.
- Now draw these diagonal lines.
- Now we can draw the final lines.

Drawing 3D Shapes on Isometric Paper

Locus

Locus (plural Loci) refers to a set of points which all share the same condition.

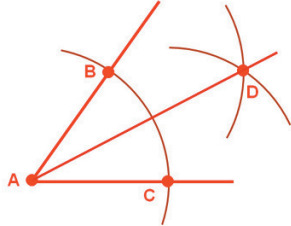
Example: Draw the locus of a point which is always the same distance from these two lines.

This means: Bisect the angle.

LOCUS FROM TWO LINES

- Set a pair of compasses to a sensible position, and keep them set at this position.
- Place compasses at **A** and draw an arc as shown, crossing both lines of the angle.
- Place compasses at **B** and draw an arc as shown.
- Place compasses at **C** and draw an arc as shown.
- Join points **A** and **D** to bisect the angle.

Answers to locus questions should be drawn as accurately as possible.



Locus from two lines

Shape: Drawing 3D Shapes on Isometric Paper

Shape: Locus

Metric Units: Length, Mass and Capacity

Questions involving these units require that you know the conversion factors.

CAPACITY

m^3 cm^3 l ml

$1\text{l} = 1000\text{ml}$

$1\text{l} = 1000\text{cm}^3$

$1\text{m}^3 = 1000000\text{cm}^3$

So: $1\text{ml} = 1\text{cm}^3$

Example 8: A can of lemonade is 330ml . What is this in litres?

Divide by 1000 Capacity = $330 \div 1000 \text{ l}$

$$\text{Capacity} = 0.33 \text{ litres}$$

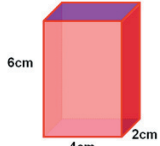
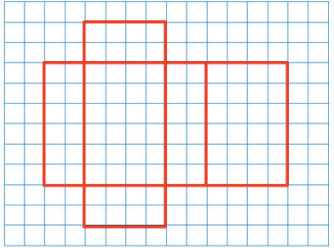
Metric Units: Length, Mass and Capacity

Drawing Nets of Solids

When drawing a net we should be as accurate as possible.

Net for a Cuboid

Example: This cuboid has sides of lengths 6cm , 4cm and 2cm . Accurately draw the net for this cuboid.

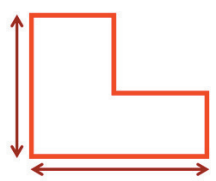
- First draw the back which is a rectangle 6cm by 4cm .
- Then draw the sides which are rectangles 6cm by 2cm .
- Then draw the top and bottom which are rectangles 4cm by 2cm .
- Then draw the front which is a rectangle 6cm by 2cm .

Nets of Solids: Drawing Nets

Shape: Metric and Imperial Units - Length, Mass and Capacity

Shape: Nets of Solids

Perimeter of an L Shape



In an L shape, the two horizontal lengths add up to the same length as the base.

Also, the two vertical lengths add up to the same length as the height.

So to find the total perimeter, we can add the base and height then double this.

Example 1: Find the perimeter of this L shape.

19 cm

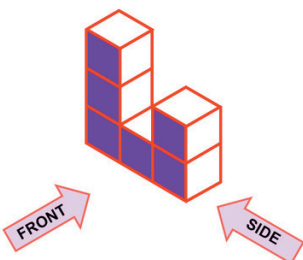
14 cm

$$\begin{aligned} \text{Perimeter} &= (14 + 19) \times 2 \\ &= 33 \times 2 \\ &= 66 \text{ cm} \end{aligned}$$

Perimeter of an L Shape

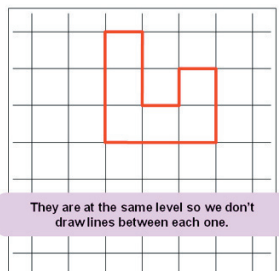
Plans and Elevations

Example: Draw the plan, front elevation and side elevation for this solid made from 6 cubes.



FRONT ELEVATION

If we look at the front we will see the six squares as shown.



They are at the same level so we don't draw lines between each one.

Plans and Elevations

Shape: Perimeter of Rectangle and L Shape

Shape: Plans and Elevations

Trigonometry in 3 Dimensions

Example: Find the size of angle AGE in this cuboid.

Draw the triangle AGE

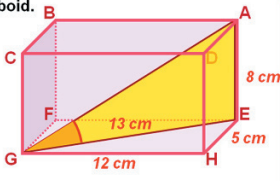
To find angle AGE
We need to find length GE, using Pythagoras' Theorem on triangle EGH:

Put numbers into Formula $GE = \sqrt{12^2 + 5^2}$

Square the values $GE = \sqrt{144 + 25}$

Add these numbers $GE = \sqrt{169}$

Square root $GE = \sqrt{169} = 13 \text{ cm}$



We can now find the size of angle AGE using Trigonometry

Pythagoras and Trigonometry in 3D

Shape: Pythagoras and Trigonometry in 3D

Pythagoras' Theorem

Sometimes a description of a situation involving a right angled triangle may be given. This may be accompanied by a diagram - if not you should draw your own.

Example 2: A rectangle has a diagonal of 24 cm. If one of the sides is 18 cm, how long is the other side?

Square the longest side $24 \times 24 = 576$

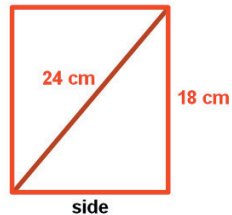
Square the other side $18 \times 18 = 324$

Subtract these numbers $576 - 324 = 252$

As you're finding one of the shorter sides

Square root $\sqrt{252} = 15.9$

side = 15.9 m



Pythagoras' Theorem: Worded Questions

Shape: Pythagoras' Theorem

Similar Shapes

Similar shapes are shapes where one shape is an enlargement of the other.
ie: All corresponding angles are the same, and corresponding sides are in the same ratio.

Similar Triangles

Example 1: Explain why triangles MNO and OPQ are similar.

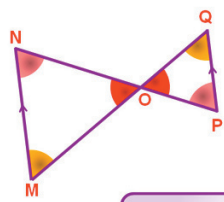
We don't have any side lengths, so we need to show that both triangles have the same 3 angles. (Note the parallel lines in the diagram).

Angles MON and POQ are opposite angles, so they are equal.

Angles MNO and OPQ are alternate angles, so they are equal.

Angles NMO and OQP are alternate angles, so they are equal.

Each triangle has the same 3 angles, so they are similar.



Similar Shapes

Shape: Similarity and Congruence

Calculating the Time

Example 2: A cyclist cycled for 15 miles at a constant speed of 12 miles per hour. For how long was the cyclist cycling?

Time = $\frac{\text{Distance}}{\text{Speed}}$

Put numbers into formula $\text{Time} = \frac{15}{12}$

Work out $15 \div 12$

Time = 1.25 h

Change 0.25 hours into minutes by multiplying by 60

$0.25 \times 60 = 15$

Write time in hours and minutes

Time = 1 hour 15 minutes

Speed, Distance and Time: Calculating the Time

Shape: Speed, Distance and Time

Volume of a Sphere

We have a formula to find the Volume of a Sphere.

$$\text{Volume} = \frac{4}{3} \times \pi \times r^3$$

Example 3: Find the radius of a sphere with volume of 56 m³.

$$\begin{aligned} 56 &= 4 \div 3 \times 3.14 \times r^3 \\ 56 &= 4.81667 \times r^3 \\ \frac{56}{4.81667} &= r^3 \\ 11.626 &= r^3 \\ \sqrt[3]{11.626} &= r \\ 2.265 &= r \end{aligned}$$

Use $\pi = 3.14$

$$\text{Radius} = 2.27 \text{ m (to 3sf)}$$

Spheres: Volume

Line Symmetry

When we reflect a shape in a mirror line, each point on the shape must be the same distance on the other side of the mirror line.

METHOD

First put two marks on the mirror line.

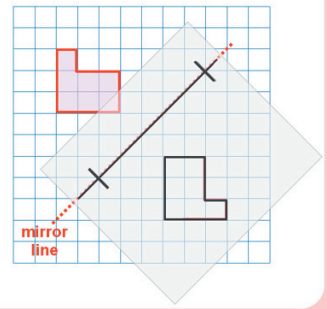
Using tracing paper, trace the shape.

Trace the mirror line and marks.

Turn your tracing paper over, lining up your mirror line with the marks.

Trace your reflection onto the page.

Example 2: Reflect the shape in the mirror line.



Line Symmetry: Reflecting a Shape

Shape: Spheres: Surface Area and Volume

Shape: Symmetry

Tessellating Triangles

ANY triangle will tessellate with triangle tiles of the same size and shape.

Let us try to see why this is true.

Take any triangle.

Make a copy of this triangle.

Rotate the copy through 180° and put them next to each other.

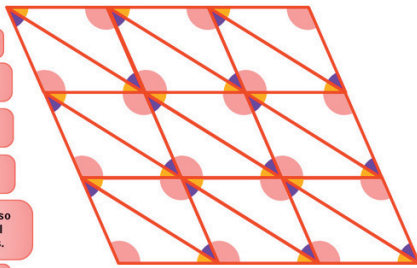
Put other pairs of triangles next to them.

The three angles of the triangle meet at a point.

These three angles add up to 180°, so whatever the triangular tile, we will always be able to put them in rows.

We can keep adding rows.

This works for any triangular tile.



Tessellations: Triangles

Shape: Tessellations

12 and 24 Hour Clock

Times can be given in either format.

12 Hour Clock

Hours are from 1 to 12.

Must say:
a.m. for morning times
and
p.m. for afternoon times.

12 o'clock mid-day is written as:

12:00 pm

12 o'clock mid-night is written as:

12:00 am

24 Hour Clock

Hours are from 0 to 23.

Time must be a
4 digit number.

12 o'clock mid-day is written as:

12:00

12 o'clock mid-night is written as:

00:00

12 and 24 Hour Clock

Shape: Time

(4) Enlargements

Using a Centre of Enlargement

A centre of enlargement tells us the position on the page in which the enlargement must be drawn in relation to the original object.

We multiply the distance from the centre of enlargement to a point on the shape by the scale factor to get the position of the corresponding point on the new shape.

Example 1:

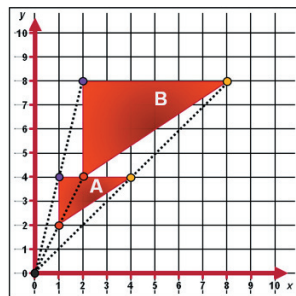
Enlarge shape A
By SCALE FACTOR 2

With Centre of Enlargement (0, 0)

Method

Find the distance from centre of enlargement to a point, then multiply this by 2.

Repeat this for the other points.

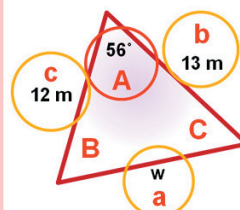


Enlargements

Shape: Transformations

The COSINE Rule

Example 1: Find the length of side w



First label the triangle*

*Hint: When you have only one angle, label it A

Next identify the values you are working with.

We are working with all three sides and an angle, so we use the COSINE RULE.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

We replace the letters in the rule with the values from the triangle:

$$w^2 = 13^2 + 12^2 - 2 \times 13 \times 12 \times \cos 56$$

Square the values and work out the multiplication:

$$w^2 = 169 + 144 - 312 \times \cos 56$$

Using a calculator: $\cos 56^\circ = 0.559$
So we replace this in the formula:

$$w^2 = 169 + 144 - 312 \times 0.559$$

$$w^2 = 169 + 144 - 174.408$$

$$w^2 = 138.592$$

Finally square root:

$$w = 11.77 \text{ m}$$

Trigonometry: Non-Right Angled Triangles

Shape: Trigonometry - Non-Right Angled Triangles

Trigonometry: Finding Sides

Example 5: Find length i .

Label the sides in relation to the angle.

Identify which sides you are working with.

Choose the formula containing the two sides you are working with.

$\sin A = \frac{\text{Opp}}{\text{Hyp}}$

$\sin A = \frac{\text{Opp}}{\text{Hyp}}$

Rearrange

$\text{Hyp} \times \sin A = \text{Opp}$

Substitute

$67 \times 0.956 = i$

Multiply

$i = 64.1 \text{ cm}$

This is the Sin of 73° . Found using a calculator.

Trigonometry: Right Angled Triangles

Multiplying a Vector by a Scalar

A scalar is just another word for a number.

To multiply a vector by a scalar, we just multiply both numbers in the vector by the scalar.

Example 2:

Find $3w$

$$w = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$3w = 3 \times \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \times 4 \\ 3 \times 1 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ 3 \end{pmatrix}$$

We can show this on a diagram.
 Multiplying a vector by 3 gives a vector of **treble the length**, in the **same direction**.

Vectors

Shape: Trigonometry - Right Angled Triangles

Shape: Vectors